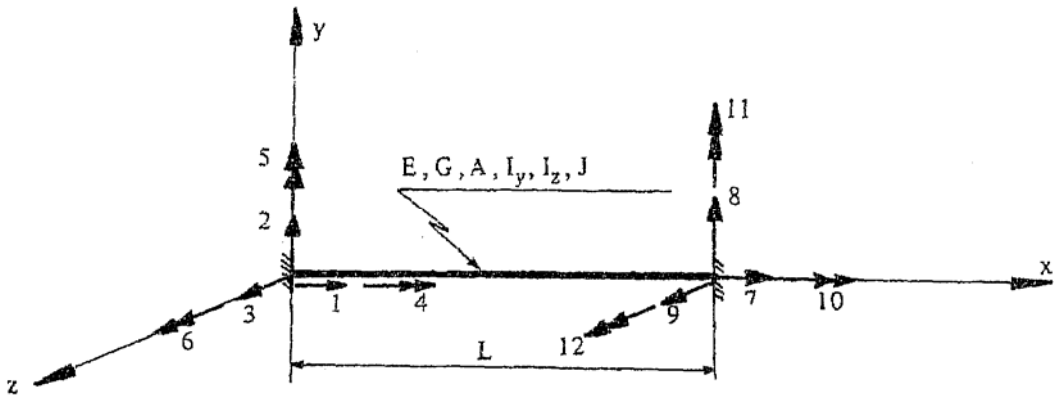
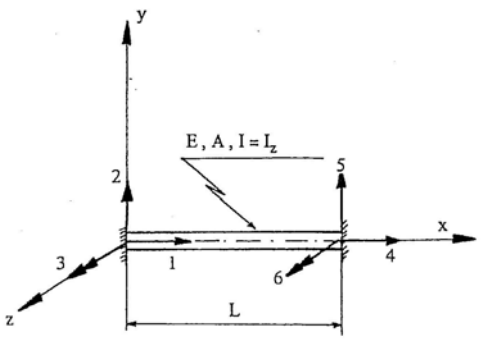
Priestorový prútový prvok



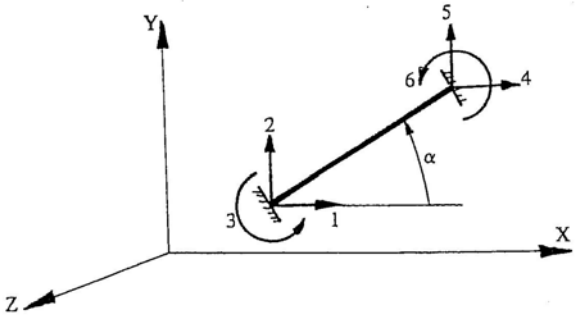
$$[k] = \begin{bmatrix}
k_{i1} & k_{i2} & k_{i3} & k_{i4} & k_{i5} & k_{i6} & k_{i7} & k_{i8} & k_{i9} & k_{i10} & k_{i11} & k_{i12} \\
\frac{EA}{L} & 0 & 0 & 0 & 0 & 0 & \frac{-EA}{L} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{12EI}{L^3}z & 0 & 0 & 0 & \frac{6EI}{L^2}z & 0 & \frac{-12EI}{L^3}z & 0 & 0 & 0 & \frac{6EI}{L^2}z \\
0 & 0 & \frac{12EI}{L^3}y & 0 & \frac{-6EI}{L^2}y & 0 & 0 & 0 & \frac{-12EI}{L^3}y & 0 & \frac{-6EI}{L^2}y & 0 \\
0 & 0 & 0 & \frac{GJ}{L} & 0 & 0 & 0 & 0 & 0 & \frac{-GJ}{L} & 0 & 0 \\
0 & 0 & \frac{-6EI}{L^2}y & 0 & \frac{4EI}{L}y & 0 & 0 & 0 & \frac{6EI}{L^2}y & 0 & \frac{2EI}{L}y & 0 \\
0 & \frac{6EI}{L^2}z & 0 & 0 & 0 & \frac{4EI}{L}z & 0 & \frac{-6EI}{L^2}z & 0 & 0 & 0 & \frac{2EI}{L}z \\
\frac{-EA}{L} & 0 & 0 & 0 & 0 & 0 & \frac{EA}{L} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{-12EI}{L^3}z & 0 & 0 & 0 & \frac{-6EI}{L^2}z & 0 & \frac{12EI}{L^3}z & 0 & 0 & 0 & \frac{-6EI}{L^2}z \\
0 & 0 & \frac{-12EI}{L^3}y & 0 & \frac{6EI}{L^2}y & 0 & 0 & 0 & \frac{12EI}{L^3}y & 0 & \frac{6EI}{L^2}y & 0 \\
0 & 0 & 0 & \frac{-GJ}{L} & 0 & 0 & 0 & 0 & 0 & \frac{GJ}{L} & 0 & 0 \\
0 & 0 & \frac{-6EI}{L^2}y & 0 & \frac{2EI}{L}y & 0 & 0 & 0 & \frac{6EI}{L^2}y & 0 & \frac{4EI}{L}y & 0 \\
0 & \frac{6EI}{L^2}z & 0 & 0 & 0 & \frac{2EI}{L}z & 0 & \frac{-6EI}{L^2}z & 0 & 0 & 0 & \frac{4EI}{L}z
\end{bmatrix}$$

Rovinný prútový prvok obojstranne votknutý



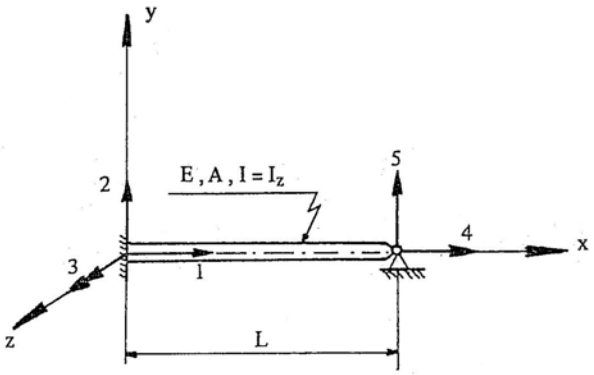
$$[k] = \begin{bmatrix} k_{i1} & k_{i2} & k_{i3} & k_{i4} & k_{i5} & k_{i6} \\ \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12\,EI}{L^3} & \frac{6\,EI}{L^2} & 0 & -\frac{12\,EI}{L^3} & \frac{6\,EI}{L^2} \\ 0 & \frac{6\,EI}{L^2} & \frac{4\,EI}{L} & 0 & -\frac{6\,EI}{L^2} & \frac{2\,EI}{L} \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12\,EI}{L^3} & -\frac{6\,EI}{L^2} & 0 & \frac{12\,EI}{L^3} & -\frac{6\,EI}{L^2} \\ 0 & \frac{6\,EI}{L^2} & \frac{2\,EI}{L} & 0 & -\frac{6\,EI}{L^2} & \frac{4\,EI}{L} \end{bmatrix}$$

$C = \cos\alpha$
$S = \sin\alpha$

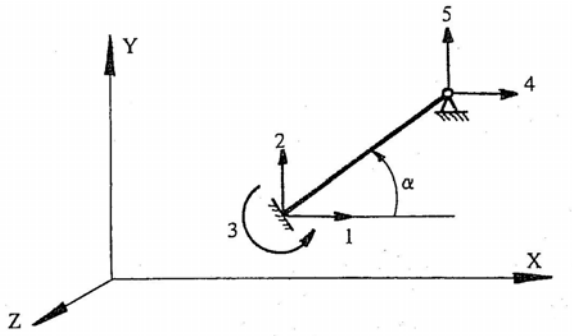


$$[k] = \begin{bmatrix} k_{i1} & k_{i2} & k_{i3} & k_{i4} & k_{i5} & k_{i6} \\ \frac{EA}{L}C^2 + \frac{12\,EI}{L^3}S^2 & \frac{EA}{L}CS - \frac{12\,EI}{L^3}CS & -\frac{6\,EI}{L^2}S & -\frac{EA}{L}C^2 - \frac{12\,EI}{L^3}S^2 & -\frac{EA}{L}CS + \frac{12\,EI}{L^3}CS & -\frac{6\,EI}{L^2}S \\ \frac{EA}{L}CS - \frac{12\,EI}{L^3}CS & \frac{EA}{L}S^2 + \frac{12\,EI}{L^3}C^2 & \frac{6\,EI}{L^2}C & -\frac{EA}{L}CS + \frac{12\,EI}{L^3}CS & -\frac{EA}{L}S^2 - \frac{12\,EI}{L^3}C^2 & \frac{6\,EI}{L^2}C \\ -\frac{6\,EI}{L^2}S & \frac{6\,EI}{L^2}C & \frac{4\,EI}{L} & +\frac{6\,EI}{L^2}S & -\frac{6\,EI}{L^2}C & \frac{2\,EI}{L} \\ -\frac{EA}{L}C^2 - \frac{12\,EI}{L^3}S^2 & -\frac{EA}{L}CS + \frac{12\,EI}{L^3}CS & \frac{6\,EI}{L^2}S & \frac{EA}{L}C^2 + \frac{12\,EI}{L^3}S^2 & \frac{EA}{L}CS - \frac{12\,EI}{L^3}CS & \frac{6\,EI}{L^2}S \\ -\frac{EA}{L}CS + \frac{12\,EI}{L^3}CS & -\frac{EA}{L}S^2 - \frac{12\,EI}{L^3}C^2 & -\frac{6\,EI}{L^2}C & \frac{EA}{L}CS - \frac{12\,EI}{L^3}CS & \frac{EA}{L}S^2 + \frac{12\,EI}{L^3}C^2 & -\frac{6\,EI}{L^2}C \\ -\frac{6\,EI}{L^2}S & \frac{6\,EI}{L^2}C & \frac{2\,EI}{L} & \frac{6\,EI}{L^2}S & -\frac{6\,EI}{L^2}C & \frac{4\,EI}{L} \end{bmatrix}$$

Rovinný prútový prvok s kĺbom v pravom uzle

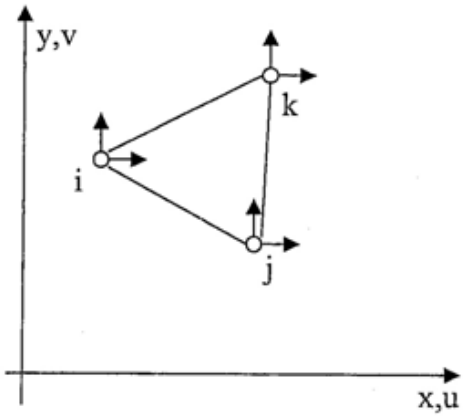


$$[k] = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 \\ 0 & \frac{3EI}{L^3} & \frac{3EI}{L^2} & 0 & -\frac{3EI}{L^3} \\ 0 & \frac{3EI}{L^2} & \frac{3EI}{L} & 0 & -\frac{3EI}{L^2} \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 \\ 0 & -\frac{3EI}{L^3} & -\frac{3EI}{L^2} & 0 & \frac{3EI}{L^3} \end{bmatrix}$$



$$[k] = \begin{bmatrix} k_{i1} & k_{i2} & k_{i3} & k_{i4} & k_{is} \\ \frac{EA}{L}C^2 + \frac{3EI}{L^3}S^2 & \frac{EA}{L}CS - \frac{3EI}{L^3}CS & -\frac{3EI}{L^2}S & -\frac{EA}{L}C^2 - \frac{3EI}{L^3}S^2 & -\frac{EA}{L}CS + \frac{3EI}{L^3}CS \\ \frac{EA}{L}CS - \frac{3EI}{L^3}CS & \frac{EA}{L}S^2 + \frac{3EI}{L^3}C^2 & \frac{3EI}{L^2}C & -\frac{EA}{L}SC + \frac{3EI}{L^3}SC & -\frac{EA}{L}S^2 - \frac{3EI}{L^3}C^2 \\ -\frac{3EI}{L^2}S & \frac{3EI}{L^2}C & \frac{3EI}{L} & \frac{3EI}{L^2}S & -\frac{3EI}{L^2}C \\ -\frac{EA}{L}C^2 - \frac{3EI}{L^3}S^2 & -\frac{EA}{L}CS + \frac{3EI}{L^3}CS & \frac{3EI}{L^2}S & \frac{EA}{L}C^2 + \frac{3EI}{L^3}S^2 & \frac{EA}{L}CS - \frac{3EI}{L^3}CS \\ -\frac{EA}{L}CS + \frac{3EI}{L^3}CS & -\frac{EA}{L}S^2 - \frac{3EI}{L^3}C^2 & -\frac{3EI}{L^2}C & \frac{EA}{L}CS - \frac{3EI}{L^3}CS & \frac{EA}{L}S^2 + \frac{3EI}{L^3}C^2 \end{bmatrix}$$

Stenový trojuholníkový prvok v kartézskych súradniciach



Aproximácia

$$u(x,y) = \alpha_1 + \alpha_2 x + \alpha_3 y$$

$$v(x,y) = \alpha_4 + \alpha_5 x + \alpha_6 y$$

Vektor deformačných parametrov

$$\{r\} = \{u_i, v_i, u_j, v_j, u_k, v_k\}$$

Vektor premiestnení

$$\{u\} = [N]\{r\}$$

$$[N] = \begin{bmatrix} N_i & 0 & N_j & 0 & N_k & 0 \\ 0 & N_i & 0 & N_j & 0 & N_k \end{bmatrix}$$

$$N_i(x,y) = \frac{1}{2A}\left(x_j y_k - x_k y_j + y_{jk} x + x_{kj} y\right), \text{ kde } \quad y_{jk} = y_j - y_k$$

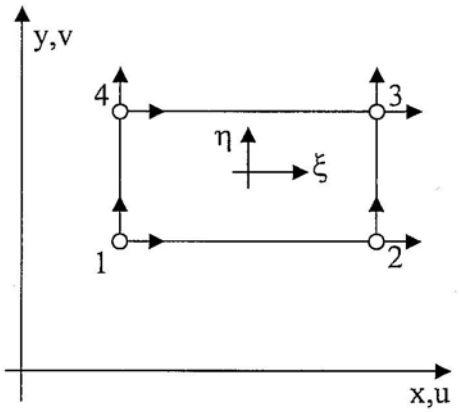
Geometrické rovnice

$$\{\varepsilon\} = [B]\{r\}$$

Tvarová matica pomerných deformácií

$$[B] = \frac{1}{2A}\begin{bmatrix} y_{jk} & 0 & y_{ki} & 0 & y_{ij} & 0 \\ 0 & x_{kj} & 0 & x_{ik} & 0 & x_{ji} \\ x_{kj} & y_{jk} & x_{ik} & y_{ki} & x_{ji} & y_{ij} \end{bmatrix}, \quad \text{kde} \quad (y_{ij} = y_i - y_j)$$

Izoparametrický stenový štvoruholníkový prvok



Aproximácia

$$u(x,y)=\sum_{i=1}^{4}N_i(\xi,\eta)u_i \qquad v(x,y)=\sum_{i=1}^{4}N_i(\xi,\eta)v_i$$

$$x=\sum_{i=1}^{4}N_i(\xi,\eta)x_i \qquad y=\sum_{i=1}^{4}N_i(\xi,\eta)y_i$$

Vektor deformačných parametrov $\{r\}=\{u_1,v_1,u_2,v_2,u_3,v_3,u_4,v_4\}$

Vektor premiestnení $\{u\}=[N]\{r\}$

$$[N(\xi,\eta)]=\begin{bmatrix}N_1 & 0 & N_2 & 0 & N_3 & 0 & N_4 & 0\\ 0 & N_1 & 0 & N_2 & 0 & N_3 & 0 & N_4\end{bmatrix}$$

$$N_i(\xi,\eta)=\frac{1}{4}(1+\xi_i\xi)(1+\eta_i\eta) \quad \forall \quad i=1,4$$

Geometrické rovnice $\{\varepsilon\}=[B]\{r\}$

Tvarová matica pomerných deformácií

$$[B(\xi,\eta)]=[B_1 \quad B_2 \quad B_3 \quad B_4], \qquad [B_i]=\frac{1}{A}\begin{bmatrix}\xi_i h(1+\eta\eta_i) & 0\\ 0 & \eta_i b(1+\xi\xi_i)\\ \eta_i b(1+\xi\xi_i) & \xi_i h(1+\eta\eta_i)\end{bmatrix}$$

kde rozmery prvku sú $L_{12}=L_{34}=2b$, $L_{23}=L_{41}=2h$, $A=4bh$ $\forall$ $i=1,4$